

Comment

The importance of the constructal framework in understanding and eventually replicating structure in tissue
Comment on “The constructal law and the evolution of design in nature” by Adrian Bejan and Sylvie Lorente

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One of the most promising techniques in regenerative medicine is the controlled growth of living and functioning tissue in the lab and the implantation of such material in a diseased organ. Actually, many of the most common medical conditions involve exactly the degeneration or reduction in numbers of cellular populations, with a corresponding inability of the body to replenish these populations and recover functionality. From chronic conditions (like ischemic myocardial heart disease) to incidental damage (like spinal cord injury), the possibility to implant successfully adequately large and appropriately functional tissue seems like a much desired option, and very substantial progress has been made in this direction [5].

The main reason we cannot culture substantially large tissue elements *in vitro* yet is the inability of prevailing physical transport mechanisms to supply nutrients and oxygen to the cells that are located in the core of such cultured samples, i.e., cells that are away from the surface: Diffusion, an omnipresent and metabolically cost-free mechanism has a limited effective range. Nature has solved this problem by utilising a finely balanced convection–diffusion duality, the specifics, origin and details of which are described nicely by the constructal law [1]. In effect, rapid transport networks (arteries) allow for distributing material efficiently to the smaller scales, where diffusion is effective and takes over. The constructal theory shows where the threshold for each process lies, in each system, [3]. It is this combination exactly, in ratios that evolution has optimised (and the constructal law sheds light on), that makes multicellular life possible.

We believe that, having such a powerful explanative tool as the constructal law opens great vistas for going beyond observation and towards design. The multifaceted processes that marry transport with biology [2] can now be viewed in a different, fully rational, light and the great questions of morphogenesis [6] can be explored not only as a phenomenon that happens but also as a process that can be replicated.

The main point the author would like to convey in this short Comment is that progress achieved to-date in tissue engineering laboratory techniques, as far as vascularisation is concerned and beyond, [4], can be harvested in a novel way by incorporating the first principles approach that the constructal law conveys. We can now think, for the first time, in terms of actively encouraging tree-like vascular growth *in the right proportions and dimensions of each*

system (using the appropriate combination of mechanical and biological cues and growth factors) in cultured tissue, since the originating principle that gives shape and form to this growth is known. There is of course a long way to go before the appropriate combinations of mechanochemical stimuli are found. The constructal framework however offers a platform within which the right questions (biochemical, mechanical and more importantly combined) can be asked and the appropriate experiments designed. Progress in this field can really evolve into a transformative medical technology that will have very substantial impact in the quality of life of millions.

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Comment

The physics principle of the generation of flow configuration Comment on “The constructal law and the evolution of design in nature” by Adrian Bejan and Sylvie Lorente

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Design – configuration, architecture, shape, structure, pattern, rhythm – is omnipresent in nature [1–4]. Bejan and Lorente’s article [5] presents the view that the generation of design in nature is a phenomenon that can be based on a principle – the constructal law. The vast majority of natural systems are non-equilibrium thermodynamic systems, containing internal flows (e.g., mass, heat, charges, goods, etc.) with a defined configuration. The first law of thermodynamics states that energy is conserved in these systems as it flows through them [5], while the second law determines that the entropy outflow must be greater than the entropy inflow. The constructal law [5] complements the other two by stating that, for a system to persist, configuration must morph (evolve) towards designs that make it easier for currents to flow. This last self-standing law of design indicates that the acquisition of configuration is a dynamic process that guides and facilitates flowing.

The constructal law is particularly simple but it appears to be ubiquitous in Nature, combining universality, simplicity and complexity all into a single design principle. The intraspecific variability of configurations inside similar systems, for instance, is an expression of the constructal design [2,6,7]. Bacterial colonies that cope with hostile environmental conditions develop branched configurations, while colonies that enjoy environments loaded with nutrients develop a compact shape instead [7]. Stony corals may develop branched or spherical shapes in a differentiated response to the variability of environmental conditions, such as the easiness of nutrient transport [6]. Pedestrians typically prefer to move freely. But in crowded spaces or when a stationary crowd stands in their way and needs to be overcome, pedestrians naturally organize themselves into streams (lanes) [8]. All these designs are anticipated by the constructal law because natural systems are not purposeless – they have objectives, functions to fulfill – and so develop designs that represent the most competitive configuration for survival. A spherical massive volume is a more effective way to fill space and extract nutrients from the surrounding environment [6,7]. But in a low-nutrient environment (or a hard agar surface or in a crowded open space) the formation of branches – bio-lanes, bio-streams, bio-rivers, or rivers of people – provide paths of lower resistance that enable coral and bacteria to thrive inside the nutrient rich region or pedestrians to penetrate the crowds [6–8].

Countries are also complex systems that are far from equilibrium because they are ‘alive’ [9]. Prominent areas – the hubs of thinkers, makers and traders –, pump flows of energy, information and goods across the country. Night-time satellite images show that while these flows present a more massive configuration of brightly lit areas in developed

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countries (i.e., resource-rich environments), they lead to a substantially more branched configuration of lights (light-lanes, light-streams) in underdeveloped ones [9]. Similarly to what happens in natural systems like corals or bacteria, the configurations of human activity appear to be optimally designed to fulfill their purpose, given the local constraints of economic development.

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Comment

Constructal Law: From law of physics to applications and conferences

Comment on “The constructal law and the evolution of design in nature” by Adrian Bejan and Sylvie Lorente

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Professors Bejan and Lorente have written an excellent review of the field defined and driven by the Constructal Law. They explain why the Constructal Law deserves the status of law in physics, i.e. the same status as the first and second laws of thermodynamics.

Beginning with the universal presence of design in nature, the authors show that the Constructal Law and the global design of nature constitute a unified view of evolution, i.e., design modifications in time, happening naturally. They also present new design features that emerge from this view. Sveltiness, for example, plays an important role in understanding the origin of the organ size. Furthermore, skeleton and vegetation are designs that facilitate the flow of stresses. As vascular designs can be seen everywhere, several examples of dendritic flow systems in nature and engineering are discussed showing that “the few large and the many small” should flow together to make better and better flow constructs over areas and volumes.

History has shown that a law is stated in words first, and that the mathematical formulation comes later. In this sense, the Constructal Law agrees with this chronology: it was stated in 1996 [1] and its complete mathematical formulation was revealed in 2004 [2].

The applications of Constructal theory are extremely numerous. The optimization of fins and assemblies of fins using constructal design has proved to be an efficient approach to discovering geometries with increasingly better performance. Geometries evolving from Y-shaped [3] to complex assemblies of fins [4,5] are examples of how the increase of degrees of freedom can lead to improvements in the performance of flow systems. On the other hand, cavities are spaces between fins and have been optimized by Constructal design from the more elemental cavities [6–8] to complex cavities [9,10].

The growing interest in the Constructal Law has led to six international conferences so far, which documented its advance and its role in Thermodynamics:

2006, Constructal Theory of Social Dynamics, Duke University

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2007, Constructal Theory of Social Dynamics, Duke University

2008, Constructal Human Dynamics, Security and Sustainability, Evora, Portugal

2008, Shape and Thermodynamics, Florence, Italy

2009, Constructal Theory and Multi-Scale Geometries, Paris, France

2010, Constructal Law Symposium, Design and Nature 2010, Pisa, Italy

The next Constructal Law Conference will be held in Porto Alegre, RS, Brazil on 1–2 December 2011 [11].

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Comment

Constructal law in technology, thermofluid and energy systems, and in design education

Comment on “The constructal law and the evolution of design in nature” by A. Bejan and S. Lorente

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It has been said many times in one form or another that there is nothing more understandable and practical than a simple theory. As engineers and scientists, we are always looking for a simple theory, law or equation in every engineering science to explain behaviours and to use as design tools. Examples of such simple theories are Bernoulli's law in fluid mechanics [1], the first and second laws in thermodynamics [2], Fourier's law in conduction heat transfer [3] and Newton's law for convective heat transfer [4].

Design, on the other hand, has been viewed by many as the subject (I am specifically not using the word “science”) which is an art in which the engineering sciences are creatively synthesised and where industry experience is preferable and in many cases essential. It may involve considerable research, thought, modelling, interactive adjustment and redesign. What it surely is not – is the application of a simple theory.

Because of the lack of industry experience, many university professors find it challenging to teach design to engineering students. Optimisation methods [5] have, however, been developed recently with many objective functions, which now make it possible to conduct designs [6–8] that rely on optimisation procedures. The objective functions now include not only “engineering science parameters” such as dimensions, temperatures and heat transfer rates, but also parameters that quantify parameters such as economics, safety, aesthetic, manufacturability, maintainability and impact on the environment. It shows that design is in many cases not an art anymore and that very good designs can be produced by people with very little industry experience and that optimum designs should be driven by a simple law. However, all these designs are not about the time direction of the “movie” of design generation and evolution and it is not about optimally end design, destiny or entropy. The concept that constructal law defines in physics is “design” (configuration) as a phenomenon in time.

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The outstanding review by Bejan and Lorente [9] shows how constructal law, since its discovery in 1996, by Bejan [10], has distinguished itself from many others in that it is not limited to physics or specifically thermodynamics or heat transfer. It is applicable to many other fields outside the natural sciences such as geography, the design of human movement, wealth and communication. What it does where many other equations, laws or theories fail is to unite all fields including design. Constructal law is a theory about the time direction of global optimisation and it explains in a simple manner the shapes that arise in nature. Flow architecture comes from a principle of maximisation of flow access, in time, and in flow configuration that is free to morph. This theory is unlike the belief that nature is fractal, and it allows engineers to analyse systems with boundary conditions under constraints in a pursuit to optimality.

Applications where constructal law was successfully used in technology, thermofluids and energy systems and in the education of design are with the design of microchannels to be used in the cooling of heat sinks [11], obtaining an increase in the heat transfer rate density by using wrinkled entrance regions [12], multiscale plate assemblies [13], pin-fins [14], microchannel heat sinks [15], rotating cylinders in cross-flow [16] and the thermodynamic optimisation of the integrated design of solar thermal cycles [8]. The road to future advances with the constructal law is wide open.

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Comment

The law of life: The bridge between Physics and Biology

Comment on “The constructal law and the evolution of design in nature” by A. Bejan and S. Lorente

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Evolution, growth and dynamics of living systems (animate, inanimate and human beings) are governed by biological principles. The living systems adopt various shapes and configurations for easy access and comfort for daily activities. These systems with various size, shapes, etc., have gradually evolved since the beginning of universe. They vary in sizes starting from insects, birds, animals to human beings. The shape and structure of various organs change to have easy access to live and to survive comfortably following a less resistive path of life. The evolution, growth and flow of life was biologically governed by Darwin’s five laws [1] which are as follows:

- Evolution as such: World is not constant and unlike mathematically invariant entities such as squares, triangles and circles, biological species are not invariant.
- Multiplication of species: Species decompose into daughter species or they may bud off into different types of descendent.
- Natural selection: The relatively few individuals who survive due to particularly well adopted combination of inherited characteristics, give rise to the next generation. The environment in which plants and animals reproduce is defined by the world around each plant or animal, and the selection occurs within a species even when the rest of the world does not change.
- Gradualism: Evolutionary change occurs through gradual change of populations and the time period for a significant change is very very long (1000 years, 10,000 years and so on).
- Common descent: Every living entity in the world belongs to a common ancestor. Ecologists further explain ‘animals’ and plants’ as ways of surviving and expanding their world.

Darwin’s rules are indeed the biological foundations of evolution, construction and growth of living systems. Now, the questions are: why does bird have low weight, fish, a streamlined body and horse, four legs? The biological evolution for various living systems may be justified by the law of construction based on Physics. Laws of Physics or principles of physical phenomena were not the basics for explaining laws of biological evolutions of Darwin, nor did Darwin explain five laws based on any physical principles [1]. Darwin’s rules on ‘natural selection’ and ‘Gradualism’

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may be regarded as the starting point of Physicists to explain and expand beyond Darwin's rule. Starting from a growth of any animate or inanimate species to its dynamics and life in a generalized framework of Physics, which is a constructal path, path of easy access, flow, morph, evolution, is the law of life, the law of construction [2].

Bejan and Lorente [3] explain the law of life as the constructal law which governs the universal phenomenon of generation and evolution of design that involves configuration, shape, structure, pattern, etc. The idea of constructal law is simple, but that may not be trivial based on classical laws of Physics. The building blocks of constructal law was discovered as a solution for art of cooling via using high conducting path in heat generating systems with fixed volume and constant heat generation rate [4]. The solution is obtained as sets of optimization and organization steps, which are termed as 'paths' or 'channels'. An elemental optimal path is termed as 'first construct' and subsequent paths are termed as 'second construct', 'third construct' and so on. It is shown that the paths form a tree-like network, in which every single geometric detail is determined theoretically [4]. Further, Bejan [4] established that the 'constructal path' concept is useful to devise efficient cooling system for electronic systems, computers etc (inanimate systems). Now, the question is why does one need to remove extra heat within electronic systems? The electronic systems also like to have easy access of flow of current via a less resistive path. The resistance of wires or components increase with temperature during various processing operations within electronic systems. Larger resistance further causes large heat generation which may deteriorate systems (inanimate). The higher rate of heat removal via *tree shaped constructal path* would enable the system to operate in a comfortable atmosphere. Bejan's constructal concept provides a solution for maintaining an optimal atmosphere with ease of operation of systems and the optimization is constrained with feedback based on the fact that the system temperature should not exceed a specified fixed temperature.

Now, Bejan's theoretical foundation of 'Constructal Law' based on principles of heat transfer and Physics to explain the optimal environment of inanimate electronic system is the key step to explain the *natural design* of living systems in a colony of Biological systems, as mentioned in an earlier note [5] which refers to an architecture of honeybee swarm consisting of several thousand of bees exposed to various temperature levels of atmosphere. Several thousand honeybees form swarms in order to regulate the 'core' or central temperature, which does not exceed 35 °C irrespective of cold/hot ambient temperature [6]. Heinrich [6] investigated a series of swarms consisting of several thousands of bees which control the internal configuration and optimal shape of the swarm such that they have a suitable environment to live. At higher ambient temperatures, the swarm consists of several vertical channels connected with each other like tree-branches which are in accordance with constructal tree network as proposed by Bejan [2,4,7]. No matter whether the transport is within animate or inanimate systems, the natural phenomena based on constructal concept [2,4] is invariant. As seen in the swarm of bees which can be treated as a dynamic system, the anatomy of swarm evolves in such a way that it provides easier access to the imposed (global) currents that flow through it [6,7]. In a similar manner, the construction of river basins with flowing water, pattern of mud cracks on the ground, and dichotomous branching in the air passage of human lung [2] are the consequences of *Constructal law* which cannot be explained with simple Physics or Biological intuition, rather with complex optimization problem using feedback from ambience, system, compatibility or many others and the solution of the entire problem is obtained with *Physics* or *Thermodynamics* as explained in the review paper [3].

Bejan and Lorente's [3] analysis on *law of life* may be regarded as a bridge between Physics and Biology. The review paper [3] laid the foundation for 'design in nature' starting with the new law of Physics (Constructal law). This new law defines the design structure which is natural for animate or inanimate systems (geophysics to biology). Bejan's new idea, new Physics on *Constructal law* is about a complex Physics which defines that the design in nature is not static, but is dynamic and the design evolves with time as *movie* in a direction which allows currents to flow easily. *This movie never ends, it evolves with time for more and more easy access, that is the movie of the entire world, that is the way world moves, civilization goes – a new bridge between Physics, Biology and Society.* The bridge between Physics and Biology has been discussed with various steps starting from *law of design during growth*, subsequently *law of structural design during evolution*, *size of organ as a part or whole of the species*, *comfort or compatibility design based on flow of stresses*, *hierarchical and vascular designs with complexities and overall design mechanisms* via a unique law, *Constructal law* [3].

The law of design for evolution of natural process, animate or inanimate species and human body are based on several common principles [3]. A few of them are based on (i) minimum entropy generation and maximum efficiency for engineering and biological systems; (ii) minimum flow resistance for heat, fluid and mass transfer involving engineering, river basin and physiology; (iii) minimum effort or cost in social dynamics, animal design and many more [3].

Bejan and Lorente [3] explained evolution as time dynamics of design which forms the basis of structural design during evolution. Structural design is not only based on explicitly Physics, Thermodynamics or Engineering principles, but with a feedback of environmental impact, system response, easy access or comfortability. *The design on evolution is evolution of configuration in time toward unreachable equilibrium flow structures that flow more and more easily through their finite size (i.e. surviving) environments* [8]. The path of animals, path of river-basin, life of man, family, country, economy can be easily explained based on the unique design of evolution. The design of structure for fliers, runners and swimmers for various species or animals can be biologically reasoned with Darwin's rule of evolution; however, *Constructal law* has profound basis on the intricate design and the predictions agree with all known speed-mass data for fliers, runners and swimmers [9,10].

No matter whether the species is a flier, runner or swimmer, the shape and construction of various organs supporting the functioning of daily activities have also been established based on *Constructal law*. The review paper [3] explains: “*The larger the organ, the less constrictive its flow passages, and consequently the work W_1 spent on driving the flow is smaller. At the same time, the work W_2 spent to carry the organ increases.*” Bejan and Lorente [3] applied principles of Physics, fluid mechanics along with constructal theory to explain the evolution of wings, muscles, blood vessels, etc., with various design parameters (length, breadth, width, etc.). For example, the concept of design of the blood vessel is unique and is based on the pumping power of the blood flow, the weight of blood vessel per unit length and the power spent to transport the vessel. Finally, the overall design was established with an optimization problem based on the minimization of the objective function as the sum of losses (pumping power of the flow and power spent to transport the vessel) [3].

The flow of stresses play a critical role and hence, *constructal law* justifies the *materials of construction*, whether muscle or bone should be the appropriate materials for structures of organs. Here is the principle of construction: *the design facilitates the flow of stresses such that the solid organs like bones are the lightest and strongest* [3]. In order to establish the constructal foundation of various organs, Bejan and Lorente [3] analyzed the detailed design of wheels and constructal design also justifies where more materials are needed within wheels to withstand the flow of stress. Based on the construction of wheel, the animals can be regarded as a rolling body. Subsequently, the design principles of first, second, third or fourth leg are established. In fact, flow of stresses are also based on *behavioral constraints* of systems. What are these *behavioral constraints*? For example, a human has two speeds: walk and run, whereas the horse has three speeds: walk, trot and gallop. The entire design problem is quite complex and can indeed be solved with the good predictions of *biological evolutions*. The design for legs to jump or crawl or run can be adequately explained. The flow of stresses can also be explained within inanimate systems. Why do the old big trees have huge thick lower tapered portion, which are the good source of wood? In fact, the design of trees is very complicated because trees have to survive against wind flow, storm, drought, rain and thunderstorms. Thus the design of organs depends on versatile constraints such that the optimal solution gives proper justification of the survival of trees with evolution of various organs.

Hierarchy plays a major role in the design complexity of animate, inanimate systems and society which are explained with *Constructal law*. A few test studies on evolution such as the construction of river basin, city, society, etc. are adequately explained in the review [3]. In a similar manner, complex vascular design also evolves. Commonality of hierarchical and vascular design is based on the fact that *they must be stepwise more complex as they become larger and older* [3]. The large vascular body must be very complex for easy access of flow and these principles are applicable for animate (lungs, vascular tissues, etc.) and inanimate system (river basins, road network, etc.).

Finally, the global design of the *animate* or *inanimate* systems were established on the fact of ‘few large’ and ‘many small’ [3]. Why do animals or human bodies have one head? Why does river basin have one large wide river with many small branches, lakes, canals? Why is there one king in animal kingdom and why does the king or president have control of money to run a country? Why is a larger flow system efficient than smaller flow systems? The entire globe with all entities is governed by a unique law, *Constructal law*, which is the philosophy of the review article [3].

The constructal law has been used in various applications such as design of corals, bacterial colonies of plant roots [11], the architecture of lungs [12], the scaling law of river basins [13], cooling of electronics [14–18], high performance and high density heat exchangers [19–23], the design of nanofluids [24], chemical engineering equipments [25,26], fuel cells [27] and many more. The scope of research of constructal law is vast especially to gain attention in biological/medicinal science with a few still unresolved issues: (i) How to design efficient drug delivery? (ii) How fast does a specific drug move based on its viscosity, temperature, flow morphology, etc.? (iii) Can surgery be improved

with the help of constructal theory? (iv) What is the viewpoint of Constructal theory or path of growth of cancer cell? Can the growth be stopped by identifying the constructal path?

Finally, there is an open question: Will the designs of animals or humans from scratch be possible with the building blocks of *Constructal theory*?

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Comment

Go with the flow: Connecting energy demand, hydropower, and fish using constructal theory

Comment on “The constructal law and the evolution of design in nature” by Adrian Bejan and Sylvie Lorente

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In all evolved natural systems energy flows and matter cycles [1]. Until the discovery of the constructal law, there was no cogent physical theory as to why certain configurations in nature lead to more effective flow architectures. Constructal theory has added a crucial component to understanding the physics of flow systems – configuration design. Young scientists should be excited by this discovery, as it allows us to revisit and improve existing designs, and create new flow systems with informed imagination.

As an engineer working on improving the ecological state of alpine river systems affected by hydropower plants, my work has taken me to Bejan and Lorente’s ‘4th front’ [2]. The study of hydropower systems is most certainly the study of flow systems, and the hallmark of the constructal law – flow architecture, can be found everywhere.

Hydro plants are busy hubs of energy extraction, stretched across the potential landscape of the European Alps, tied together in a transnational distribution network where scheduling is based on local demand and where global economic costs are increasingly relevant. Each discharge from a reservoir creates a diurnal flood wave, a pattern of alternating crests and troughs when viewed as a daily hydrograph. Downstream of the hydro plants, hydropeaking waves surge through the river, creating a sharp increase in local hydraulic stresses on a minute-to-minute basis, and as the banks swell and velocities increase, waves ripple over the river surface, dissipating on gravelly shores. But the alpine river is not just made of rock and water. It is teeming with the chemistry of life; aquatic plants, benthic invertebrates and fish all participate in this directed and animate system. And all of the components, animate or inanimate have a constructal role to play. They work to increase the access of energy traveling through the river system, and aid in cycling the materials flowing through it. For all organisms in the alpine river ecosystem, persistence comes at the cost of participation. Fish are recognizable as individual entities in the river system because they are distinct in both their form and function; they are not bags of molecules. It is no coincidence that fish are svelte.

It has been shown that an organism’s specific entropy production decreases with increasing age [3]. It has also been proposed that individual organisms can be best viewed as spatially and temporally condensed systems of ecological processes, which via natural selection tend to increase the energy flux through their systems [4]. Evidence of the constructal law’s applicability to the evolution of animate systems can also be found when considering that

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the weight-normalized metabolism (size-independent respiration intensity) of organisms has increased exponentially over the last 500 million years [5]. The persistence and design evolution of animate systems is best described in thermodynamic terms, when they are seen as being unified in process with their surroundings. There is no ‘fish out of water.’

The constructal law provides insight when studying the animate and the inanimate, as well as systems containing both. The use of constructal theory provides a bridge across the lonely divide which often isolates physicists from ecologists, engineers from biologists. The general statement of the constructal law is powerful precisely because we are surrounded by, embedded in, and ourselves are, flow systems.

Evolution connects animate system development in time; ecology connects it in space, chemistry in composition, and constructal theory in configuration. Constructal theory’s rejection of teleology and focus on process development is a gift to the sciences which is long overdue. Future designers can invoke the constructal law when environmental mitigation is considered by striving to design energy extraction systems whose distinguishing mark is their directed persistence, flow architectures with evolving access and deep interactivity between the animate and the inanimate. Such designs can bestow young scientists with a fresh and inherently optimistic, not optimal view of science. The architects of the future have much useful work to do, whether physicist, biologist, engineer, or economist. We cannot beat Carnot, but designing with constructal theory may help us get that much closer.

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Comment

Design in nature, and the laws of physics
Comment on “The constructal law and the evolution of design in
nature” by Adrian Bejan and Sylvie Lorente

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Every day we get surprised and amazed by the huge variety of shapes and sizes of natural systems. We call it the “natural beauty” of the “wonderful mother nature” and this is nothing more than a way of expressing our inner feeling that some order must exist behind the natural scenery. Our brain perceives it as “beauty”, which might be the first stage of a deeper and unifying perception that involves rationalization based on mathematics and principle.

The previous lines might serve as a preface to the paper “The constructal law and the evolution of design in nature” [1] which is a comprehensive review in which the authors summarize recent applications of the Constructal Law. The results and examples provided by the authors show how a new principle sheds new light on old problems, and how it provides a new unified understanding of complex systems.

After many years of research on shape and structure of natural systems, I am pretty sure that the “Constructal Law” put forward by Adrian Bejan [2] provides the rationale for understanding the evolution of shape and structure in a unifying way in both natural and manmade systems. It is a fact that Constructal Law has been successfully applied to describe and anticipate the unique features of a wide variety of systems [3–5]. Now, we are able to perceive natural design as Nature fine art that has a rationale behind, such that our new feeling of “natural beauty” comprises also the scientific dimension of beauty. On the other hand, the Constructal Law makes it outdated and useless the “Theory of intelligent design” put forward by some neo-creationists (see [6]).

The Constructal Law is as general as the First and Second Laws of Thermodynamics but has a very different scope that makes it unique and complementary to those laws. While the First Law points to conservation of energy, both the Constructal Law and the Second Law point to change, i.e. to a direction in time. Though both these laws share this common feature, they diverge as with respects to the scope. Contrarily to the Second Law, Constructal Law applies to systems out of equilibrium, i.e., systems that evolve in time. While the Second Law deals with state variables, the Constructal Law combines flows and design (size, shape, structure).

Constructal Law provides the foundations for both natural and engineered design to be viewed in the light of science. It opens new avenues of research in many fields from engineering and geosciences to life and social sciences. The reason for this ubiquitous applicability of the Constructal Law is rooted in the idea that what is important is not “what flows”, but how inner flows shape and structure the systems in an evolutionary way that points to better and

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better performance. In this sense, Constructal Law defines a direction in time. “Being” (which is timeless, and does not transform) and “becoming” (temporal and evolutionary) are two philosophies that express complementary visions of the world. While the First Law is the Physics principle behind the concept of “being”, both the Second Law and the Constructal Law are the scientific counterpart of the Philosophy of the “becoming”.

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Comment

Universality of design and its evolution
Comment on “The constructal law and the evolution
of design in nature”

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Design (configuration, geometry, hierarchy, pattern, rhythm, shape and structure) occurs everywhere: in the biological systems we belong, in the ecological and economical systems we live, and in the technical systems we utilize. Its origin and evolution always fascinate all of us and have constantly attracted efforts in understanding and predicting them [1–8]. Because of the complexities of the problem, however, attempts have relied on approaches of a more or less ad-hoc or empirical nature. These conventional approaches are generally limited in application to particular designs and usually have a narrow range of validity.

The constructal theory offers a revolutionary new approach for understanding and predicting the designs that arise everywhere, in animate, inanimate and engineered systems. According to the theory, the phenomenon can be accounted based on a single principle (the constructal law): “*For a finite-size flow system to persist in time (to live) it must evolve in such a way that it provides greater access to the currents that flow through it*”. In their review [9], Bejan and Lorente have beautifully demonstrated how this law governs all animate and inanimate design and evolution, how this law confirms and corrects the views and findings from the other ad-hoc approaches and prevalent in the literature and society, and how this law offers revolutionary views and predictions of design and its evolution.

The first and the second laws of thermodynamics define energy and entropy, respectively, the two important thermodynamic state properties [10–12]. These two concepts and their beautiful features have facilitated applications of the two first principles significantly [10–13]. The analogy inspires the question regarding the existence of a new state property that comes from and will offer a powerful tool to support applications of the constructal law. It would be wonderful to see some progress towards the answer to this question.

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Comment

The Constructal law: From design in nature to social dynamics
and wealth as physics
Comment on “The Constructal law and the evolution of design
in nature” by Professor Adrian Bejan and Professor Sylvie Lorente

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Professors Bejan and Lorente wrote an excellent, rare and very timely paper [1]. It is a “must read” for all who work in research and universities. “The Constructal Law and the Evolution of Design in Nature” offers a new revolutionary approach based on physics for understanding and predicting the designs that arise in nature, from the oneness of animate and inanimate designs, the origin of finite-size organs on animals and vehicles, the flow of stresses as the generator of design in solid structures (skeletons, vegetation), and the global design of human flows. The physics principle that governs the phenomenon of generation of configuration in nature is the Constructal Law: “For a finite-size flow system to persist in time (to live), its configuration must change in time such that it provides easier and easier access to its currents (fluid, energy, species, etc.)”

The impact of the Constructal law is significant already, and the present paper describes it very well. This manuscript is an extremely timely review of the Constructal-law contributions made recently, in the last five years. In particular, we like the accessible level of the presentation, suitable for a diverse audience. Profs. Bejan and Lorente are, without any doubt, the best qualified to describe the state of this fast growing field.

We are astonished by the breadth of the territory covered with the Constructal law. We were aware, as authors and researchers, of the extensive Constructal work on the geometric optimization of shapes for cooling electronics (trees of several kinds, see Refs. [2–17]), but we did not know about all these much, much bigger implications here treated, among which:

- biology, the Constructal Law was used to explain the design of corals, bacterial colonies and plant roots, the architecture of lungs, the heat transfer in the circulatory system, and many features of dendritic flow architecture in vascular design;

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- geophysics, the Constructal Law was offered as theoretical basis for plate tectonics, beach sand and slope, the scaling laws of river basins and the evolution of Constructal morphology in all of nature;
- social dynamics, the Constructal Law has inspired an activity that offers a physics foundation for phenomena of pattern and emergence.
- Economics, the Constructal Law accounts for the relationship between movement and economic activity, and makes “wealth” a concept in physics.

With this, it seems to us that Constructal theory has united biology with physics, and has removed the distinction that we thought existed between physics and engineering. Now every design in nature is “physics”, and, based on the evidence amassed in this review, the Constructal Law is a law of physics.

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