

The physics of spreading ideas

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Abstract

Here we show that the main features of citations history (S-shaped curve, increasing h index, decreasing m quotient) are predictable with the constructal law of how an idea flows over a populated territory. It flows in two ways: fast and long, as “convection” along established channels of researchers interested in the idea, and slow and short, sideways from channels, by “diffusion” to new users. The first regime accounts for the rising portion of the S curve, and the second for the slow-down. Tree-shaped established channels spread the idea faster than single channels over the same area. The complexity of the design of fast channels increases in time due to the evolution of communications technologies.

Keywords: Constructal law, Design in nature, S curve, h index, m quotient, Invasion and consolidation, Convection and diffusion.

Nomenclature

A	area, m ²
b, c	constants
h	h index, Eq. (2)
k	constant
L _i	lengths of tree branches, m, Fig. 7
L ₁	length of tree trunk, m, Fig. 7
m	m quotient, Eq. (8)
n	number of length scales of tree structure
t	time, s
V	rate of publishing, articles/ year
x	number of publications
x ₀	constant

y number of citations

Greek symbols

α diffusivity, m^2/s

δ boundary layer thickness, m

Subscripts

co consolidation (diffusion)

in invasion (convection)

Introduction - Measuring scientific performance

Scientific publishing is a competitive activity. Creative individuals think, work, and write to have fun intellectually. Along the way, they have impact on their disciplines and contribute to society and to themselves.

The peer groups and institutions that govern this activity have developed several measures for how a scientific author is performing. Two decades ago and earlier, the obvious measures were “numbers”—the number of publications and the number of citations, annual and lifetime. These numbers are expectedly greater when the career is longer and more established, and consequently they tend to obscure the authors who have impact early in their careers. To get around the age bias, scientific publishing has shown a preference for two new measures, the h index and the m quotient [1]. These two numbers are used extensively, from tenure and promotion decisions to the granting of society awards.

In this paper we show that the h index is defined in such a way that it must increase with age. Likewise, the m quotient is designed such that it must decrease during most of the career. The h is greater for the old, and the m is greater for the upstart. This is useful to know, especially in competitive academic circles. Much more important is the physics of why age influences both h and m.

Here we show that the age effect is fully predictable because it is a consequence of the S shape of the history of citations acquired by a single publication, single author, or single institution. We also show that the S-shaped history is predictable. It is a manifestation of the natural design of spreading a new idea on an area inhabited by potential users of the new idea.

The S shape of citations history

The number of citations of a single scientific article increases in time as a function that resembles the logistics S curve (Fig. 1) [2-10]. So does the number of citations of all the articles of one author. The physics argument of why this curve *must* have the S shape is presented in Section 5. Here we begin with why the S shape is responsible for the age effect on h and m .

The scientific publishing and citations history of a single author is shown qualitatively in Fig. 2. The author publishes new articles at a certain rate during the most productive phase of the career. Each paper generates a number of citations that is a S-shaped curve versus time (e.g. Fig. 1). Some articles are better than others, and they tend to level off at higher numbers of life time citations. This is particularly evident in the comparison between single-idea articles and review articles, as in the two examples of Fig. 1, Ref. [4] versus Ref. [5]. The summation of all these numbers of citations is itself an S-shaped curve of total number of citations versus time, which tends toward a cumulative plateau.

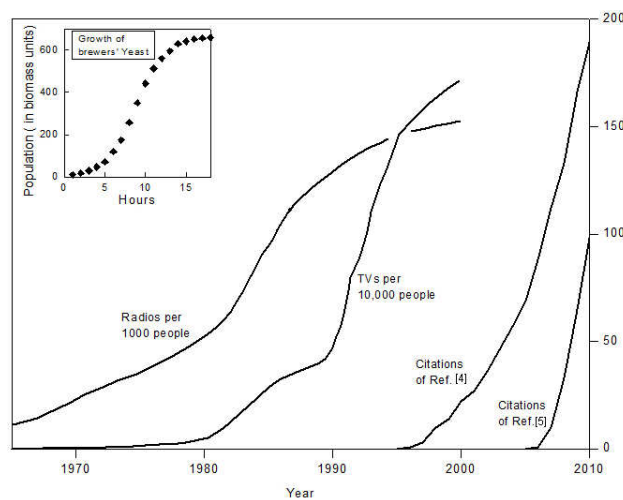


Figure 1

To make our argument transparent analytically, we assume that the publishing career of the author consists of publications that have the same S-shaped curve, and are published at equal time intervals: n articles per year (Fig. 3). This is a simplified model of one publishing career (Fig. 2), however, the lifetime effect of the model is the same as that of the real publishing career—it is an S-shaped curve that rises the fastest during the most productive period of the

author's career.

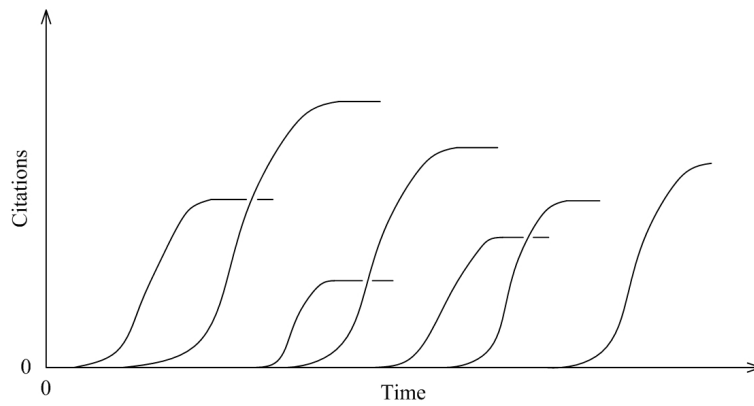


Figure 2

The advantage of using the simple model is shown in Fig. 4. The individual S curves fall on a single S curve when the start of each curve is placed at $x = 0$. The abscissa counts the articles produced during the author's career to the point (t_1) indicated by the oldest article, which is the most cited and the highest ranked (no. 1). Its abscissa location (x_1) is proportional to its age (t_1), and it moves to the right at constant rate,

$$x = Vt \tag{1}$$

where $V = n$ articles/year (Fig. 3).

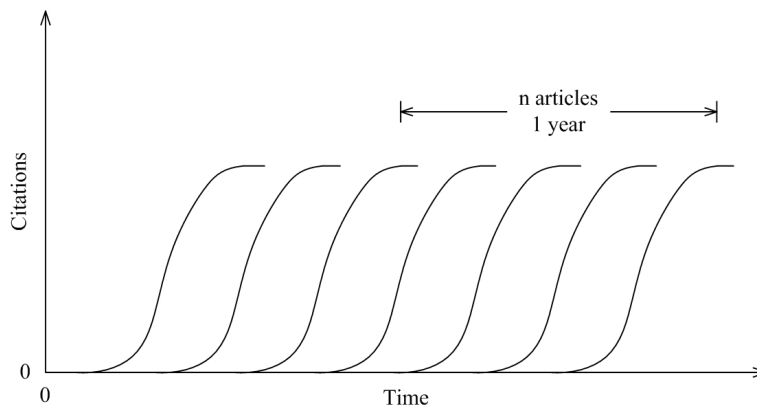


Figure 3

The h index. If the S shape of the curve in Fig. 4 is represented by the function $y(x)$, then the h index is defined by the point where

$$y(x-h) = h \quad (2)$$

The rate at which h increases is obtained by differentiating Eq. (2) with respect to time,

$$\frac{dh}{dt} = \frac{y'(x-h)}{1 + y'(x-h)} V \quad (3)$$

where $y' = dy/dx > 0$, and $y'(x-h)$ is the slope of $y(x)$ at $x-h$. This means that the growth rate of h (namely dh/dt) is smaller than the growth rate of the number of the author's articles (V). The h index increases monotonically in time, along a curve $h(t)$ that has a shape similar to $y(x)$ in Fig. 4.

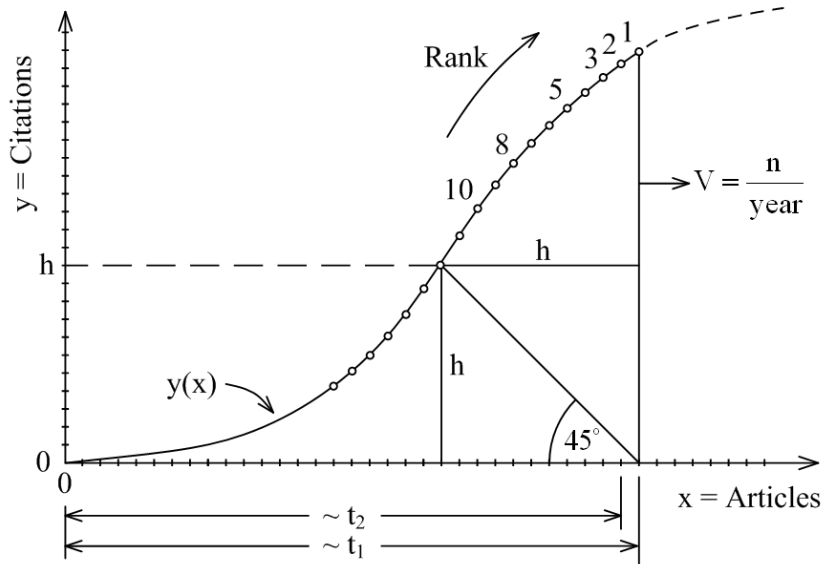


Figure 4

Toward the end of the career, when the S curves of most of the highly ranked articles have reached the plateau, the slope $y'(x-h)$ is small relative to 1, and dh/dt is small in comparison with V . Said another way, intense and repetitive publishing late in the career has little effect on the author's h index. In this limit, the h value approaches the plateau value of $y(x)$.

If the early portion of the $y(x)$ curve is approximated by the power function

$$y = b x^k \quad (4)$$

where $k > 1$, then the early portion of the $h(x)$ curve is stated implicitly by

$$x = h + \left(\frac{h}{b}\right)^{1/k} \quad (5)$$

In the beginning, x and h are small, and $h \cong b x^k$, which means that in Eq. (5) the second term dominates on the right side. When h becomes greater than $b^{-1/(k-1)}$, the first term dominates, and $h \cong x$. The late portion of the $y(x)$ curve can be approximated as

$$y = c (x - x_0)^k \quad (6)$$

where $k < 1$, and x_0 is a constant determined from matching functions (4) and (6), cf. Fig. 5a. Assuming that $x_0 \ll x$, the $h(x)$ curve that corresponds to Eq. (7) is roughly

$$x = h + \left(\frac{h}{c}\right)^{1/k} \quad (7)$$

which means that the late portion first behaves as $h \cong x$, and continues toward long times as $h \cong c x^k$.

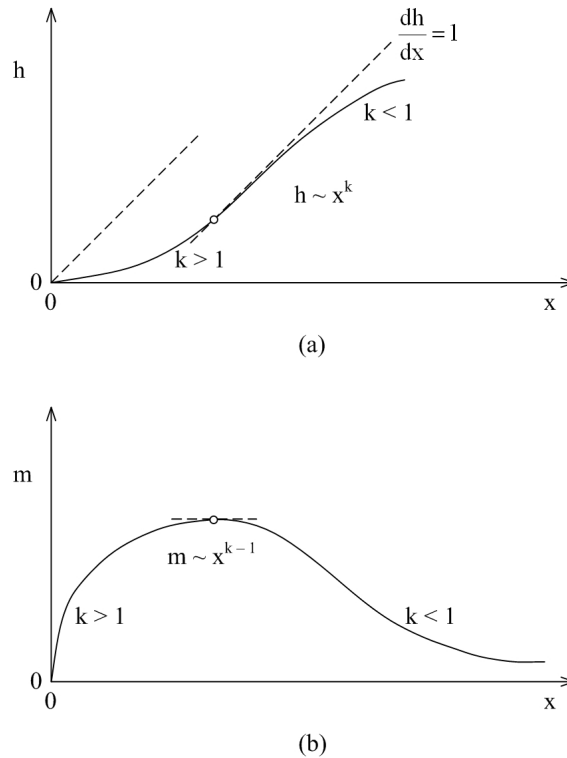


Figure 5

Together, the early and late portions account completely for the S-shaped $y(x)$ curve, in accord with the analysis provided in section 3. In the mid range where Eqs. (4) and (6) match, h increases at the same rate as x , and that rate is V . These features are summarized in Fig. 5a. The fact that Eqs. (4) and (6) merge on the line of slope $dh/dx = 1$ (and not on another tangent with a slope that depends on the assumed power exponent k) is a consequence of the h definition, Eq. (2).

The m quotient. Because the h index increases with the length of the publishing career (x), the h value tends to favor the more senior authors. To suppress this feature, Hirsch [1] introduced the m quotient,

$$m = \frac{h}{t(\text{years})} = \frac{h}{x/V} \quad (8)$$

as a “yardstick to compare scientists of different seniority”. Here we show that m fails in this mission. The explanation is provided in Fig. 5b. During the early portion of the $h(x)$ curve, Eq. (4), m increases rapidly, as x^{k-1} , where $k > 1$ [the $m(x)$ curve drawn near $x = 0$ in Fig. 5b is for cases where $1 < k < 2$]. During the late portion of the $h(x)$ curve, Eq. (6), the m value decreases as x^{k-1} , where $k < 1$.

The entire behavior of m versus time (or x) is summarized in Fig. 5b. The m quotient is constructed in such a way that it decreases during most of the productive part of the author’s career. Whereas h favors the more senior, m favors those at the start of their publishing career.

Why the citations history is an S curve

The shape of the citations curve (Fig. 1) is due to the physics of the flow of a new idea (publication, innovation, news, technology) on the landscape. Like any other point-area flow in nature [11], a new idea spreads on the landscape in two ways, via two flow mechanisms:

1. Fast, along pre-existing (established) channels of prior interest in what spreads, and
2. Slow, perpendicularly to the lines, sweeping the interstices that fill the landscape.

Both flows cover the territory inhabited by the recipients of the idea. They are the black lines (1) and the white interstices (2) of the design of how the idea flows on the area. Both flows start when the new idea becomes public ($t = 0$), however, mechanism (1) is short lived in comparison with mechanism (2), because the idea travels along the channels faster than diffusion across the interstices.

The spreading of ideas is admittedly an abstract mental viewing. To connect it to reality we propose the simple heat flow model shown in Fig. 6. This model serves several purposes. First, it accounts for the fact that the spreading of an idea is a *flow system* that connects one point with an area. The area is the square $A = 2L_1 \times 2L_1$, and is defined by the inhabitants who receive the idea. The point is the middle of the left side of the square.

Second, this flow system has dimensions that indicate its presence in space (area A , length L_1) and in time. We will show that there are two time scales: one short (t_{in}) for the “invasion” phase ruled by mechanism (1), and the other long (t_{co}) for the “consolidation” phase during which the idea sweeps by diffusion all the interstices, i.e. the entire territory.

Line invasion. The model of Fig. 6 is the simplest because it has only one pre-existing channel—drawn as a straight line of length $2L_1$ —on the square area A . More complicated, lightning-shaped paths may exist on the same territory (Fig. 7) and their flows can be analyzed in the same manner as the flow of Fig. 6.

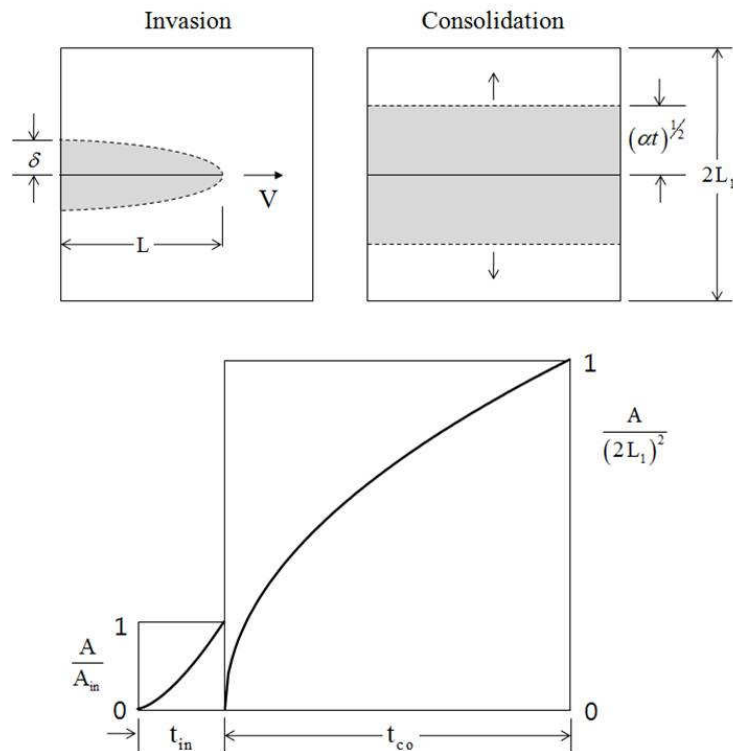


Figure 6

Imagine the release of the new idea at the point source at $t = 0$. In the reality of scientific publishing, most of these events correspond to the start of the peer review process. The first to become empowered by the new idea are those who are positioned at the entrance of the pre-existing channel of interested (established) recipients. These are the editors themselves and the referees contacted by the editors. Time passes, as the idea travels through the publication channel toward progressively more distant inhabitants of the established channels: the published journals, the email inboxes of authors connected to the journal, the shelves of the best stocked libraries, the lesser libraries that borrow from the former, and the far-distant researchers who maintain themselves in the established channel by requesting personal copies of the publication from colleagues who reside upstream in the channel.

In time, the established channel grows while the established recipients publish first-generation papers triggered by the new idea. If the time needed to reach the farthest inhabitants of this fast channel is t_{in} (the invasion time scale), then the growth of the channel line during the invasion phase has the speed scale

$$V = \frac{L_1}{t_{\text{in}}} \quad (9)$$

Older examples of this initial fast channel are simpler and better known. In our own domain (thermal and fluid sciences), we are familiar with the phenomenon of how Prandtl's boundary layer theory [12] spread on the scientific landscape one hundred years ago. It spread fast, but only to those who were connected to Prandtl: his postgraduate students (Blasius, Pohlhausen, Schlichting, von Karman) and his contemporaries who read and published in German. Later, the fast spreading continued along established channels in English, as a consequence of the American occupation of Prandtl's institute, and the American translation of Prandtl's German articles.

Today, because of electronic publishing and the web, we tend to lose sight of this pre-channeled flow architecture that exists among us, because its t_{in} scale is short and becoming shorter. Still, the first flow on the area occurs along established lines (e.g. Fig. 6), and we can all recognize it if we think of the publishing that existed before the web and much earlier. Think of the time of Galilei, Newton and Leibnitz, when the archival publications and libraries were few and slow, and the inhabitants of the established channels knew each other and communicated

their new ideas through letters carried by courier and stage coach.

The interested recipients know each other. When they look over each other's shoulder, they unwittingly collaborate. This is why the established channel is a lot faster than the body of those who are not yet connected to the established channel.

The fast flow of the idea along the channel is accompanied by diffusion of the idea sideways, to the immediate (untouched) vicinity of the channel. The first to benefit from the new idea are those who are closest to the pre-existing channels. The collaborators of the first in the channel are those who start working on the idea first, and who publish second-generation papers first. From them, and in the same vicinity, new recipients learn to apply the idea and to publish it in subsequent-generation papers. The thickness of this local territory grows by diffusion, from author to reader, and from old article to new article. In time, the "diffused" vicinity grows to be the thickest near the entrance to the fast, pre-existing channel (e.g., the upper-left of Fig. 6). This is how the established become even more established, i.e. *entrenched*.

In physics, diffusion processes are characterized by speeds of spreading that decrease in time. The reason for the monotonic decrease is that the flow encounters local inertia in its path. In thermal diffusion, for example, the heat flux that arrives at a point must raise the temperature of that point before it can proceed *diminished* to the next points downstream [12]. The vicinity swept by diffusion during the time t has the thickness $\delta \sim (\alpha t)^{1/2}$, where α is the diffusivity of the medium. The speed with which this region expands ($d\delta/dt$) decreases in time as $(\alpha/t)^{1/2}$.

This model captures the main features of the diffusion of the idea transversally to the fast channel. The body of unconnected recipients poses inertia in this transversal direction. Every new author needs time (to learn, to apply and to publish the idea) before the publication can serve as source for the next, more peripherally situated authors. Every such publication is a *diminished* source for the next. It is diminished in comparison with the publication that inspired it, which is older, with a farther reach by diffusion. The idea flows from those who know and publish to those who want to know and to publish. When the downstream recipients have risen to such a level that they have learned and published, the flow of the idea ends. Old news does not travel.

In the model of Fig. 6, the number of those who have been reached by the idea is

proportional to the finger shaped area of length $L = Vt$ and base thickness $\delta \sim (\alpha t)^{1/2}$. This area is of order $A \sim \delta L \sim \alpha^{1/2} V t^{3/2}$. At the end of the invasion phase ($t_{\text{in}} = 2L_1/V$), the diffused area has grown to $A_{\text{in}} \sim \alpha^{1/2} V^{-1/2} (2L_1)^{3/2}$, or in relative terms

$$\frac{A}{A_{\text{in}}} \sim \left(\frac{t}{t_{\text{in}}} \right)^{3/2} \quad (10)$$

As shown at the bottom of Fig. 6, Eq. (10) accounts for the early part of the S curve of Fig. 1, because the area growth rate dA/dt increases monotonically in time.

After the established channel is traveled by the idea completely ($t \sim t_{\text{in}}$), the recipients are reached by diffusion on both sides of the channel. As shown in the upper-right corner of Fig. 6, this body of recipients is represented by the area swept by diffusion during the *consolidation* phase, which lasts until

$$t_{\text{co}} \sim \frac{L_1^2}{\alpha} \quad (11)$$

Before this time, the area swept by diffusion is $A \sim 2 \times (\alpha t)^{1/2} \times 2L_1$, and its history is traced as

$$\frac{A}{4L_1^2} \sim \left(\frac{t}{t_{\text{co}}} \right)^{1/2} \quad (12)$$

This area function accounts for the late portion of the citations S curve (Fig. 1), because in Eq. (12) the growth rate dA/dt decreases monotonically in time.

The time and area scales of the invasion and consolidation phases depend on the line invasion speed and the diffusivity perpendicular to the line,

$$\frac{t_{\text{co}}}{t_{\text{in}}} \sim \frac{L_1 V}{2\alpha} > 1 \quad (13)$$

$$\frac{A_{\text{in}}}{4L_1^2} \sim \left(\frac{\alpha}{2L_1 V} \right)^{1/2} < 1 \quad (14)$$

In sum, the invasion-consolidation scenario accounts completely for the S-shaped history of citations. The S shape is not unique, because it depends on the dimensionless group $L_1 V/\alpha$. This group increases in time, because it is dominated by the speed (V) with which the established channel is traveled by the first release of the idea. From this comes the prediction that in the future t_{in} will continue to decrease relative to t_{co} , and the citations history curve will look more

like in the lower right frame of Fig. 6.

Tree invasion. The physics of the spreading of ideas over an inhabited territory is the same if the fast invasion pattern is more complicated than in Fig. 6. Consider the dichotomous tree pattern sketched in Fig. 7. This is a more realistic model of how the established recipients are connected on the territory. They are connected with *hierarchy*, which was noted already in the description of the line-shaped invasion along established channels. The recipients who benefit from the idea first are the ones who form the trunk of the tree (L_1). Subsequent branches of the tree (journals, news releases, libraries) spread the idea to the rest of the established network, as a multi-scale drawing (L_2, L_3, \dots) that covers the territory incompletely.

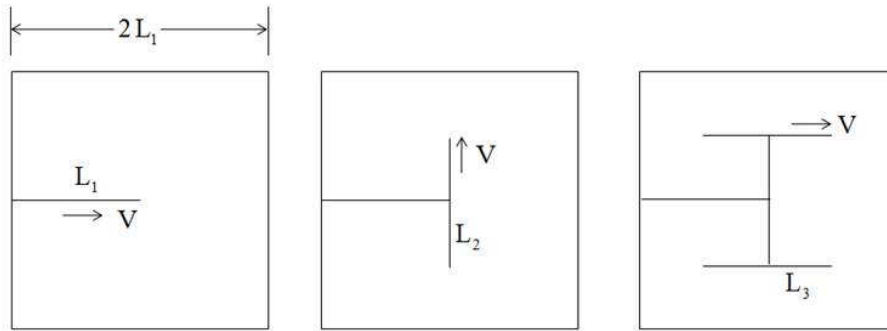


Figure 7

The analogy between the flows in Figs. 7 and 6 is made visible analytically by noting that the tree branches have lengths of size $L_1, L_2 = \frac{1}{2}L_1, L_3 = \frac{1}{2}L_1$, etc. When the number of length scales (n) is large, this sequence is approximated as

$$L_{i+1} / L_i \cong 2^{-1/2} \quad (15)$$

The idea travels from the trunk (L_1) to the farthest branches (L_2) along invasion paths of length

$$L_{in} = L_1 + \dots + L_n = L_1 \frac{1 - (2^{-1/2})^{n+1}}{1 - 2^{-1/2}} \quad (16)$$

which when $n \gg 1$ is $L_{in} \cong 3.4 L_1$. The invasion time is

$$t_{in} \cong 3.4 \frac{L_1}{V} \quad (17)$$

Slender diffused areas develop along each of the invading tree branches, in the same manner

as the $\delta \times L$ finger shown in the upper-left of Fig. 6. These fingers become wider by diffusion during the consolidation phase of their respective interstices. The consolidation phase is ruled by the time scale of diffusion over the smallest interstices ($L_n \times L_n$), which fill the territory completely. There are 2^{n-1} such interstices, with $L_n \sim (2^{-1/2})^{n-1}L_1$, therefore

$$t_n \sim \frac{L_n^2}{\alpha} \sim 2^{-(n-1)} \frac{L_1^2}{\alpha} \quad (18)$$

This time scale is comparable with t_{co} of Eq. (11), but it is shorter.

The tree-shaped invasion ends at $t \sim t_n$, when the most distant members of the established network (on branches L_n) are as remote as the unconnected recipients who fill the white interstices. For these recipients, the interstices are of size $L_n \times L_n$.

The smallest length scale of the invading tree is *finite*. Its size is deducible from the observation that the time needed by the idea to travel along the smallest channel (L_n/V) is just as long as the time needed to diffuse perpendicularly to the channel (L_n^2/α). From this time balance results $L_n V / \alpha \sim 1$, and after using $L_n \sim (2^{-1/2})^{n-1}L_1$, we obtain

$$n \cong 1 + \frac{\ln(L_1 V / \alpha)}{\ln \sqrt{2}} \quad (19)$$

The complexity (n) of the invading tree increases because the group $L_1 V / \alpha$ increases in time as a consequence of the evolution of publishing communications technologies.

Conclusions and further directions

The chief conclusion is that the main features of the spreading of ideas are predictable. The new ideas spread over a population (territory) based on two flow mechanisms, one long and fast (invasion, similar to convection) and the other short and slow (consolidation by diffusion). The predicted features are the S shape of the citations history, the increase of the h index in time, and the decrease of the m quotient after the early phase of a publishing career. Another phenomenon covered by this theory is the increasing complexity and length of the pre-existing (established channels), because of the evolution of publishing and communications.

We presented this theory in simple heat-transfer terms, by focusing on a single publishing career. The applicability of his mental viewing is considerably more general. It can be used to

account for the citations history of a group of authors, such as a university department, a research institute, a university, and a country—each as a flow of spreading ideas on territories with larger area and time scales. The same theoretical framework sustains the spreading of a single idea, such as Prandtl's boundary layer theory [12].

At the largest scale, this work casts a light on the design of the flow of talent and education, from generators of ideas to young recipients on the territory. It speaks of the flow of knowledge from the established channels to the entire population. It is the physics backbone of an invisible pattern of social organization that facilitates the flow of knowledge and governs empowerment.

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Figures

Figure 1 Examples of S-curve phenomena: the growth of brewer's yeast [2], the spreading of radios and TVs [3], and the growth of the readership of scientific publications [4, 5].

Figure 2 The pattern of publishing and citations of a single author.

Figure 3 Regular pattern of publishing by a single author.

Figure 4 Superposition of the S-curves of Fig. 3.

Figure 5 The S shape of the history of the h index, and the corresponding early hump of the m quotient.

Figure 6 Line-shaped invasion, followed by consolidation by transversal diffusion. The predicted history of spreading on the area reveals the S-shaped curve of Fig. 1.

Figure 7 Tree-shaped invasion, showing the pre-existing channels traveled by the new idea, and the interstices that will be swept by diffusion later.