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# The principle that generates dissimilar patterns inside aggregates of organisms

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ABSTRACT

Pattern formation and self-organization are phenomena that occur across the board, in animate and inanimate systems. In this paper, we rely on the constructal law to explain the generation of patterns (shapes, structures) in aggregates of organisms – pedestrian crowds and stony corals. In pedestrian crowds a variety of patterns are often observed, from 'chaotic' appearances to spontaneous organization in lanes of uniform walking direction. Stony corals and other organisms also present intraspecific variability in shape. We show that flow systems develop in time patterns which provide easier access to the nutrients and space, within a set of constraints imposed by each situation. Flow systems have the freedom to morph their shape in search for architectures that allows them to have greater access to the space that they inhabit. We identify the mechanisms allowing pedestrians to evolve in space and time. We also show that stony corals may develop branched or spherical shapes, depending on which shape performs best in response to the environmental conditions. The constructal law allows systems with complex internal flows to be described and understood for a unified view.



## 1. Introduction

The formation of dissimilar patterns inside similar systems has attracted the attention of many physicists and biologists. Despite the ubiquity of these patterns, our knowledge of how they evolve or even how they form is limited. This study addresses the formation of shapes in systems composed of aggregates of organisms such as pedestrian crowds and stony corals. Crowd dynamics rules a wide range of applications [1–3]. The common view is that human dynamics is unpredictable, because our first impression is that people move chaotically. Systematic observations of pedestrian movement revealed that in standard situations (for instance, running to catch a departing bus; panic situation excluded) there are regularities [4–9]. Pedestrians prefer to walk at the speed of least energy consumption, which is known to all of us as the comfortable walking speed. This speed depends on gender and age [10,11] and it is about 1.34 m/s, with a standard deviation of 0.26 m/s. Pedestrians prefer to move freely, but in crowded spaces the movement of individuals self-organizes naturally into lanes with specific direction. When a stationary crowd stands in the way and needs to be passed through, pedestrians organize themselves into river-like streams. Are these characteristic patterns developing by chance, or can they be anticipated from principle?

The constructal law [12] captures the phenomenon of generation of design (shape, structure, geometry) in flow systems. According to this law, in order "for a flow system to persist in time (to live) it must evolve in such a way that it provides easier access to the imposed currents that flow through it". It states that if a system is free to morph under global constraints

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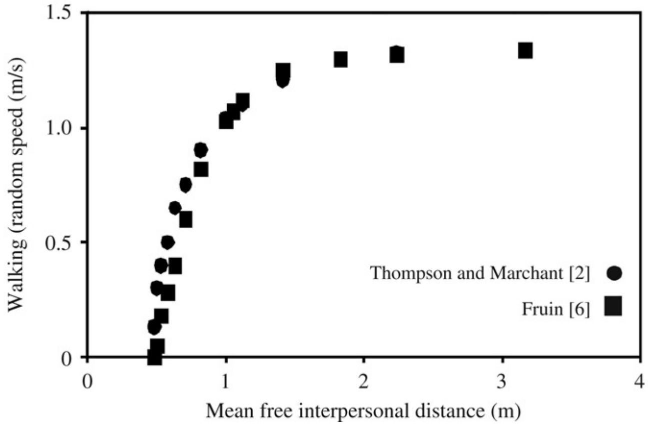


**Fig. 1.** Walking (random) speed versus mean free interpersonal distance.

(volume allocated, material properties, etc.), the architecture morphs in time in the direction of flowing more easily. The acquisition of shape is therefore an evolutionary process which is deduced from a physics principle, and not assumed in advance or postulated. There is no end to this sequence of flow images (configuration), just time direction.

## 2. Pedestrian flows: Constructal self-organization

Consider first the principle that generates dissimilar patterns in crowd dynamics. Behind a broad class of processes in the natural sciences is the dynamics that combines Brownian motion (diffusion) with some form of deterministic drift (convection). Here we shall consider a convection–diffusion equation of the general form

$$\frac{\partial n}{\partial t} + \nabla.(Un) = D\nabla^2 n \tag{1}$$

where $n$ denotes the density, $U$ the velocity and $D$ the diffusion coefficient. By applying scale analysis [12] to this equation, we obtain the time scales for diffusion ($t_d$) and convection ($t_c$),

$$t_d \sim \frac{L^2}{D} \quad \text{and} \quad t_c \sim \frac{L}{u}. \tag{2}$$

In these expressions, $u$ is the rate of drifting and $L$ is a characteristic linear dimension. The characteristic speeds that correspond to diffusion ($v_d$) and convection ($v_c$) are $v_d \sim 0.5\left(\frac{D}{t}\right)^{1/2}$ and $v_c \sim u$, respectively. This means that the initial diffusion speed $v_d$ is much larger than $v_c$, but it decreases as $t^{-1/2}$ and ends up falling under $v_c$. The time of transition from diffusion to organized flow (Eq. (2)) is $t^* \sim D/u^2$. This simple analysis shows us that diffusion is the better flow mechanism at the very beginning of the transport process, but at times greater than $t^*$, convective transport becomes the preferred mechanism.

In summary, there are two competing trends that account for the architecture of the flow system—diffusion (slow flow, or high resistivity) and streams, channeling (fast flow, or low resistivity). This combination of flow mechanisms provides greater flow access than one mechanism alone. It was used to explain inanimate designs such as turbulence and dendritic solidification (snowflakes) [12,13]. What is then the mechanism that drives pedestrian movement to evolve in space and time?

Consider pedestrian groups that proceed from one point to every point of a finite-size area (the territory). According to the constructal law, the emerging architecture will be the one that promotes the easiest flow of pedestrians. To estimate diffusion and convection time scales (Eq. (2)), we need the pedestrian diffusion coefficient ($D$) and the pedestrian walking speed ($u$). According to field surveys [6–11] the pedestrian walking speed (rate of drifting) is about 1.34 m/s. Worth noting is that pedestrians in a shopping mall or busy city street (i.e., in a random and nondirectional crowd) exhibit a speed that is related to the free area available around each individual (Fig. 1). From an analogy with kinetic theory, the pedestrian diffusion coefficient is related to the walking (random) speed ($u_r$) and the mean free interpersonal distance ($\lambda$) via the Einstein–Smoluchowski equation, i.e., $D = 0.5u_r\lambda$. The relation between the pedestrian diffusion coefficient and the mean interpersonal distance can be established with the help of this equation. On the basis of the empirical data [2,6] which are represented in Fig. 1, the relationship between $D$ and $\lambda$ is illustrated in Fig. 2. The form of the resulting curve-fitted equation is

$$D = 1.97\lambda - 0.61 \quad 0.31 \leq \lambda \leq 3.16 \text{ m} \quad \text{and} \quad L > 0.31 \text{ m}. \tag{3}$$

Why does pedestrian crowd motion exhibit dissimilar patterns, diffusion and streams? According to the constructal law the flow system has the freedom to morph in time to develop patterns that provide easier access to its currents.

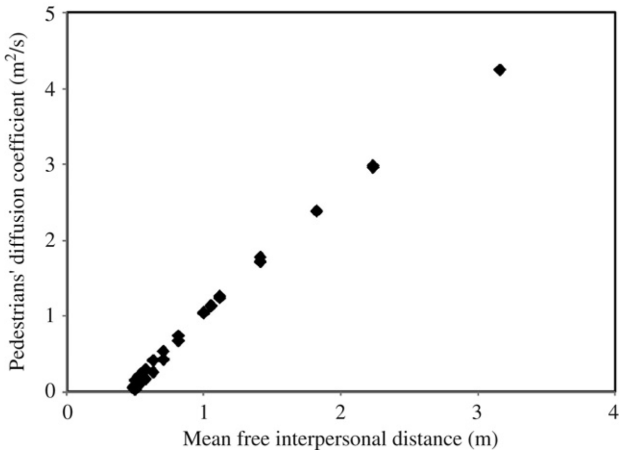


**Fig. 2.** Pedestrian diffusion coefficient versus mean free interpersonal distance.

When $t^* \sim 1.1\lambda - 0.34$, both diffusion and streams promote the easiest flow of pedestrians, but diffusion is more effective than streams if $t_{disp} < t^*$. So, diffusion is more effective when $\lambda > 0.68L + 0.31$ or $L < 1.47\lambda - 0.45$. Therefore, diffusion is more competitive for a large mean free interpersonal distance (i.e., very small densities of pedestrians) and for accessing small distances. This result is in agreement with empirical observations that pedestrians can diffuse among themselves only at very small pedestrian densities [2,6,7].

One of the more striking phenomena in pedestrian dynamics is the occurrence of spontaneous, self-organized motion. Pedestrians in very crowded open spaces tend to organize in lanes of uniform walking speed [9]. When facing a stationary crowd, pedestrians spontaneously self-organize in river-like streams ("rivers of people" [12]) in order to penetrate the crowd [9]. Why do pedestrians exhibit such self-organization? In line with the constructal law, they move in configurations that provide greater access. The flow resistivities for diffusion ($R_{spd}$) and streams/lanes ($R_{lane}$) are [14] $\frac{R_{spd}}{L} \sim \frac{\theta}{D}$ and $\frac{R_{lane}}{L} \sim \frac{\nu}{w^2}$. In these equations, $\nu$ and $\theta$ are the "viscosity" and the "driving potential" of what flows, respectively, and $w$ is the lane width. The Chapman–Enskog theory [15,16] may be employed to estimate the viscosity, $\nu \sim \frac{\lambda D}{w}$. Thus, streams/lanes provide the best flow access as long as the mean free interpersonal distance between the pedestrians obeys the following criterion: $\lambda < \frac{w^3\theta}{D^2}$. For $\theta \sim 2\ \text{m}^2/\text{s}^2$ and $w \sim 0.7$ m [5,10], the mean interpersonal distance $\lambda$ should be less than $\sim 0.8$ m. This result explains why pedestrians self-organize into lanes when their density is high enough, i.e., when interpersonal distances are small enough, as observed by Helbing et al. [9].

The occurrence of rivers of people through stationary crowds can be predicted from the constructal law in the same way as the structure of river basins [12,17]. The stationary crowd is the river basin, and the space vacated by the crossing pedestrians is the eroded river bed. Each pedestrian opens a small space toward the stationary crowd, thereby creating the conditions for a successor to follow. The next pedestrians follow those who are already in motion, giving rise to organized lines that form through the crowd. The channels due to the coalescence of such paths are tree-like structures similar to river branches.

In crowds that panic, the above formulation is not applicable because the lanes and the river-like streams of people are destroyed: individuals do not know the right way to escape. They strive to go forward, thereby reducing interpersonal distances, which might induce interpersonal contact (collision) or even loss of balance.

## 3. Self-organized patterns in coral colonies

Stony corals are also able to present intraspecific variability of patterns. They may develop branched or spherical shapes in a differentiated response to the variability of environmental conditions. Stony corals collected from exposed growth sites with stronger water currents exhibit shapes that are more spherical and massive than in corals of the same species living in sheltered sites. The latter have more thin-branched morphologies [18]. How can such different forms arise from the same "material"? Is this variability of shape necessary for survival? Branched and round massive shapes have different abilities to fill the same space, and thus a different ability to harvest the nutrients that are available. The survival of these flow systems calls for patterns that promote flow access. The preferred pattern is the one that allows the living system to deplete the nutrients as fast as possible.

Consider $s$ to be the characteristic external length scale of the living system, and $l$ the branch (needle) length scale width ($l \ll s$). The volumes of branched and spherical patterns scale as $\sim l^2 s$ and $s^3$, respectively. Because $l^2 \ll s^2$, spherical patterns are more effective for filling the space (i.e., the most effective for extracting the available nutrients). In this sense, spherical means perfection, but in reality branched patterns are also likely to occur. How can one reconcile such a contradiction?

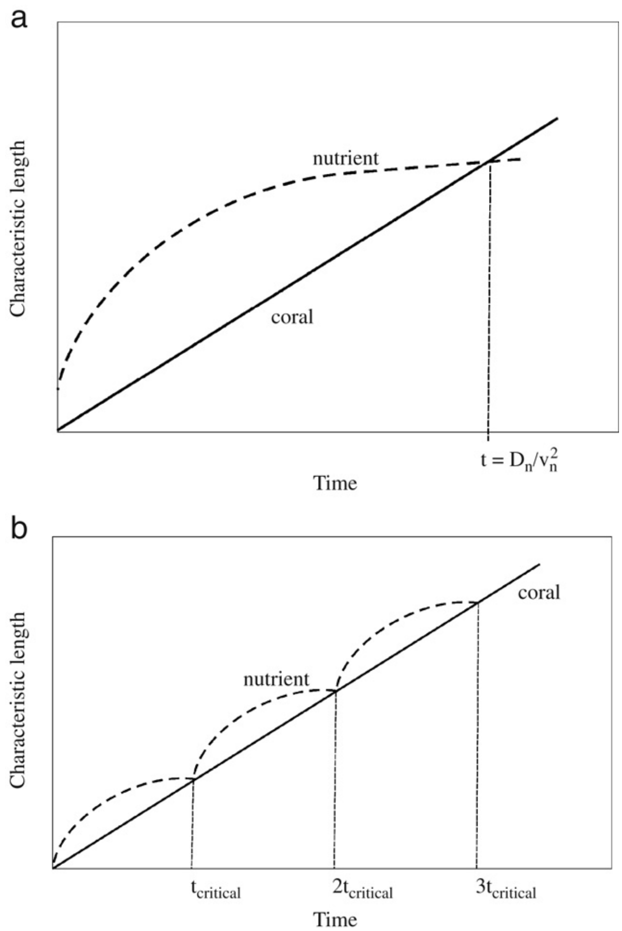


**Fig. 3.** Time evolution of characteristic dimensions of a living system and nutrients: (a) the speed of nutrient propagation falls behind the growth speed of the living system at $t > \frac{D_n}{v_n^2}$; (b) coral branches and critical times.

Consider, for example, stony corals growing at a rate $u_c$ of a few cm/year (for *Pocillopora damicornis* the growth speed is 1–6 cm/year [18]). When the coral grows in a sheltered site, where water currents are practically absent, diffusion is the most important nutrient transport mechanism. The living system starts to grow at birth ($t = 0$). Immediately after, nutrients close to the system are quickly consumed and depleted. The concentration of nutrients decreases, which triggers a wave of nutrients defined by the characteristic linear dimension $(D_n t)^{1/2}$ (Eq. (2)) and the speed of propagation $v_n \sim 0.5 \left(\frac{D_n}{t}\right)^{1/2}$, where $D_n$ is the diffusion coefficient for nutrients. The initial speed of propagation is greater than any growth speed of the living system, but decreases with the inverse of the square root of time. Consequently, the speed of nutrient propagation drops below the growth speed of the living system when $t > \frac{D_n}{v_n^2}$.

Before $t \le \frac{D_n}{v_n^2}$, the round (massive) shape is the most effective arrangement for filling the flow space. After this time scale, the coral begins to grow outside the nutrient diffusion region (Fig. 3(a)). Branches (bio-lanes, or bio-rivers) develop in order to generate low-resistance paths for nutrient access, in a phenomenon that resembles path creation in crowd dynamics. This channeling enables the system to continue to experience growth inside the nutrient rich region from $t_{critical}$ to $2t_{critical}$. At times slightly greater than $2t_{critical}$ the coral sticks out of the nutrient region. New branches grow forward in order to access the nutrient region until a new critical time is reached (Fig. 3(b)). Each branch generates a new group of branches, and the global feature of this scenario is the dendritic pattern. This pattern represents the most competitive configuration under these circumstances.

It is interesting to ask whether the size of the nutrient particles and the ambient temperature have any effect on the shape of the living system. According to the Stokes–Einstein relationship, the diffusion coefficient increases monotonically with the absolute temperature. However, the diffusion coefficient is also inversely proportional to the nutrient size. Therefore, an increase in the ambient temperature leads to a larger critical time and a less branched pattern. Conversely, an increase in nutrient size leads to shorter critical times, with the coral morphing into a more dendritic pattern.

In the case of corals growing in open sites where convective currents are significant, the water currents that transport the nutrients are much stronger than the growth speed of the coral. Therefore, the characteristic dimension of this living system is never overtaken by the characteristic dimension of the nutrient transport. The system always grows inside a region where nutrients are readily available. Consequently, the critical time is never reached and the coral develops into a round massive shape because this is the most effective arrangement for filling the available flow space in the shortest time. This explanation is backed by observations reported by several authors [18,19].

## 4. Conclusion

We invoked the constructal law to explain the formation of dissimilar patterns inside systems composed of aggregates of living organisms. On the basis of the classical convection–diffusion equation, we obtained the scales of the development of patterns that are characteristic of pedestrian crowds and stony corals. Pedestrian flows and coral colonies evolve in time as the result of the continuous search for easier flowing configurations. Patterns do not develop by chance, but result from the permanent struggle for better flowing performance when the flow configurations are able to morph in time.

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